

Math 13: Multivariable Calculus

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Placement Test

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For this course, we will be using the textbook “Calculus: Early Transcendentals” by James Stewart, 8th edition. The course covers topics in multivariable calculus including partial derivatives, multiple integrals, and vector calculus.

Contents

1 Iterated Integrals

1 Introduction to Integration, Iterated Integrals

Definition (Double Integral)

The **double integral** of a function $f(x, y)$ over a rectangular region $R = [a, b] \times [c, d]$ is defined as:

$$\iint_R f(x, y) dA = \int_a^b \int_c^d f(x, y) dy dx$$

where $dA = dy dx$ represents the area element in the xy -plane.

Definition (Triple Integral)

The **triple integral** of a function $f(x, y, z)$ over a rectangular region $R = [a, b] \times [c, d] \times [e, f]$ is defined as:

$$\iiint_R f(x, y, z) dV = \int_a^b \int_c^d \int_e^f f(x, y, z) dz dy dx$$

where $dV = dz dy dx$ represents the volume element in the xyz -space.

Definition (Iterated Integral)

An **iterated integral** is an integral that is computed in steps, where the result of one integral is used as the integrand for the next. For example, a double integral can be expressed as:

$$\iint_R f(x, y) dA = \int_a^b \left(\int_c^d f(x, y) dy \right) dx$$

If $f(x, y)$ is continuous on R , then:

$$\iint_R f(x, y) dA = \lim_{\Delta x \rightarrow 0, \Delta y \rightarrow 0} \sum_{(x_i, y_j) \in R} f(x_i, y_j) \Delta A_{ij}$$

Proposition 1.1 (Linearity of Integrals)

For any constants a and b , and continuous functions $f(x, y)$ and $g(x, y)$ on region R :

$$\iint_R [af(x, y) + bg(x, y)] dA = a \iint_R f(x, y) dA + b \iint_R g(x, y) dA$$

Proposition 1.2 (Scaling of Integrals)

If c is a constant, then:

$$\iint_R cf(x, y) dA = c \iint_R f(x, y) dA$$

Proposition 1.3

If $g(x, y) \geq f(x, y)$ for all $(x, y) \in R$, then:

$$\iint_R g(x, y) dA \geq \iint_R f(x, y) dA$$

2 Fubini's Theorem, Integration over Non-Rectangular Regions

Theorem 2.1 (Fubini's Theorem)

If $f(x, y)$ is continuous on a rectangular region $R = [a, b] \times [c, d]$, then:

$$\iint_R f(x, y) dA = \int_a^b \left(\int_c^d f(x, y) dy \right) dx = \int_c^d \left(\int_a^b f(x, y) dx \right) dy$$

Proposition 2.2

$$\iint_R g(x)h(y) dA = \int_a^b g(x) dx \int_c^d h(y) dy$$

where $R = [a, b] \times [c, d]$ and $g(x)$ and $h(y)$ are continuous functions.

Proposition 2.3

If $m \leq f(x, y) \leq M$ for all $(x, y) \in R$, then:

$$m \cdot \text{Area}(R) \leq \iint_R f(x, y) dA \leq M \cdot \text{Area}(R)$$

where $\text{Area}(R)$ is the area of region R .

- 3 Integration in Polar Coordinates**
- 4 Applications of Double Integrals**
- 5 Triple Integrals**
- 6 Cylindrical Coordinates**
- 7 Spherical Coordinates**

- 2 Vectors and Geometry of Space**
- 3 Vectors, products, planes**
- 4 Change of variables and the Jacobian**
- 5 Projections, vector functions**
- 6 Partial derivatives, directional derivatives, and the gradient**
- 7 Line Integrals**
- 8 Surface Integrals, Divergence and Stokes' Theorems**